

The Fitch Trick and Variants

ନୀଳଧରା ମିଶ୍ର • Neeldhara Misra

ସାରାଂଶ. The magician selects a volunteer to shuffle the deck and choose any five cards. The magician examines the cards, hides one of the five cards, sets the remaining four cards in a faceup row, and has the volunteer retrieve an accomplice from outside the room. The accomplice briefly examines the cards and identifies the missing card.

This is a trick that is to be performed by Aalaya (a magician) and Badarika (an accomplice) with a deck of n cards. Badarika steps out of the room and leaves her phone, smart-watches, etc. behind, so nobody can communicate with her. Then the audience picks out p cards out of a deck of n cards and gives it to Aalaya. Aalaya then arranges q out of p cards in a row¹ and Badarika comes back and takes a good look at the q cards on the table, and then identifies the $p - q$ missing card(s).

How is this possible? Specifically, for what values of n, p and q is this trick feasible?

The most common version of this trick is done with $n = 52$ (a standard deck of cards) $p = 5, q = 4$.

...image here.

¹The cards can also be placed in a stack, if the audience worries that Aalaya is signaling by angularities in the layout of the cards.

In this combination, notice that Badarika has to guess one out of 48 possible cards, since the deck has 52 cards of which four are face up. On the other hand, Aalaya can arrange the four cards in $4! = 24$ different orders, which is fewer than 48. This seems like we are falling short by a factor of two: but notice that Aalaya actually has a point of leverage that we have not accounted for: she can *choose*² the card that she wants to set aside.

So what is the right way of thinking about a signaling strategy? On the one hand, there are:

$$\binom{52}{5} = 2,598,960$$

possibilities for what can happen at the trick. Call this set $\mathcal{P}$. Now the number of ordered sequences of four cards is

$$52 \cdot 51 \cdot 50 \cdot 49 = 6,497,400;$$

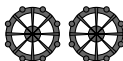
call this set $\mathcal{Q}$.

As an example, let $\alpha = \langle 4S, 5H, 2C, 7C, 6D \rangle \in \mathcal{P}$ be the subset of cards handed to Aalaya by the audience. Now Aalaya has 120 possibilities for what she can do with these cards. These possibilities are all recorded in $\mathcal{Q}$.

What we want is a way to map every element α in $\mathcal{P}$ (what the audience does) to a distinct element β in $\mathcal{Q}$ (what Aalaya would do). Now Badarika comes and sees β and maps it back to α and figures out the missing card. Can you figure out such a mapping that Aalaya and Badarika can use to perform and impress?

 Associate Professor, Computer Science and Engineering; IIT Gandhinagar

²If Aalaya is facing a tough crowd, they might not let her pick the hidden card, and might insisting on making the choice for her. Then indeed Aalaya has only 24 signals: but turns out there is a way out even in this scenario!



✉ neeldhara.misra@gmail.com

